

From fakeons and Lee-Wick models to quantum gravity

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The theory : $\zeta > 0 \quad \alpha > 0 \quad \xi > 0$

$$S_{\text{HD}} = -\frac{\mu^{-\varepsilon}}{2\kappa^2} \int \sqrt{-g} \left[2\Lambda_C + \zeta R + \alpha \left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2 \right) - \frac{\xi}{6}R^2 \right]$$

New quantization :

the would-be ghosts are turned into
fake particles ("fakeons")

A consistent projection allows us to define
the space of physical states

D. A., On the quantum field theory of the gravitational interactions,

J. High Energy Phys. 06 (2017) 086 and arXiv:1704.07728 [hep-th]

The idea is an extension of a notion that emerged from the reformulation of the Lee-Wick models

T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix, Nucl. Phys. B 9 (1969) 209;
T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033

as nonanalytically Wick rotated Euclidean theories

D. A. and M. Piva, A new formulation of Lee-Wick quantum field theory,
J. High Energy Phys. 06 (2017) 066 and arXiv:1703.04584 [hep-th]

D. A. and M. Piva, Perturbative unitarity of Lee-Wick quantum field theory,
Phys. Rev. D 96 (2017) 045009 and arXiv:1703.05563 [hep-th]

which in turn became the only possibility
to make sense of higher-derivative theories
after having shown that they are inconsistent
if formulated directly in Minkowski spacetime

U.G. Aglietti and D. A., Inconsistency of Minkowski higher-derivative theories,
Eur. Phys. J. C 77 (2017) 84 and arXiv:1612.06510 [hep-th]

because in that case they violate the

locality of counterterms



$$\frac{\ln \Lambda_{UV}}{P^2}$$

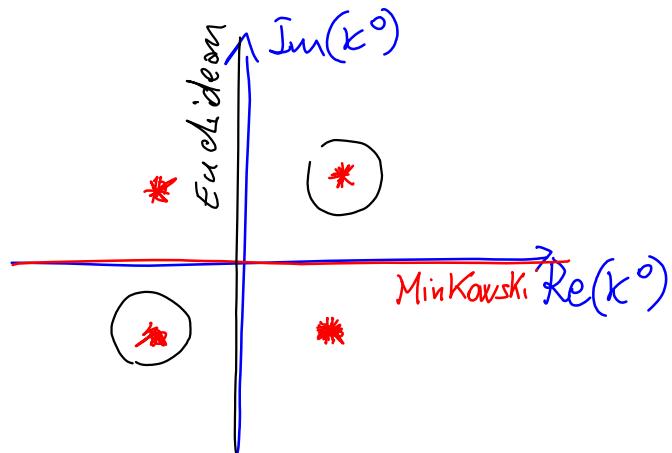
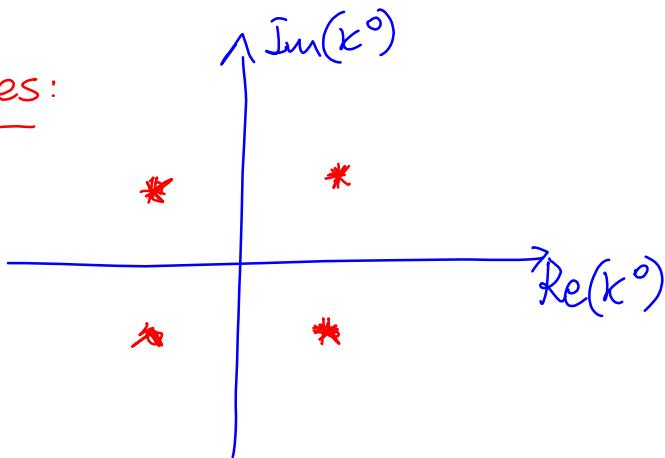
[Much worse
for triangle
& box diagrams]

Consider the HD propagator

$$S(k) = \frac{1}{(k^2)^2 + M^4} = \frac{1}{(k^2 + iM^2)(k^2 - iM^2)}$$

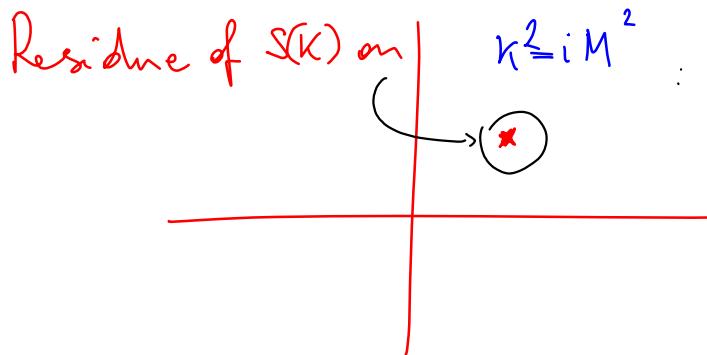
$$k^0 \in \mathbb{C}$$
$$\vec{k} \in \mathbb{R}^3$$

Poles:



Bubble diagram

$$\text{Bubble diagram} \quad \rightarrow \circlearrowleft \quad = \int_{\text{Mink}} \frac{dk^o}{2\pi} \int \frac{d^3 k}{(2\pi)^3} S(k) S(p+k)$$



$$S(k) \sim \frac{1}{\omega} \sim \frac{1}{|\vec{k}|}$$

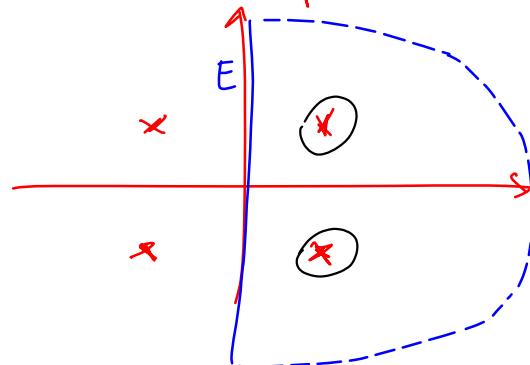
for $|\vec{k}|$ large

$$S(p+k) = \frac{1}{(p^2 + k^2 + 2p \cdot k)^2 + M^4} = \frac{1}{(p^2 + iM^2 + 2p \cdot k)^2 + M^4} \sim$$

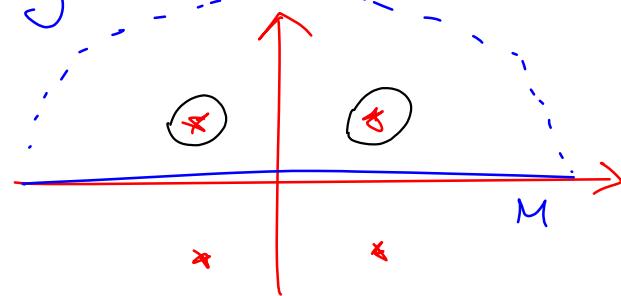
$k^2 = iM^2$

$$\sim \frac{1}{4(p \cdot k)^2} : \begin{matrix} \text{depressed power counting} \\ + \text{IR singularity} \end{matrix}$$

The nonlocal divergences cancel summing
the residues as prescribed by the Euclidean
theory :

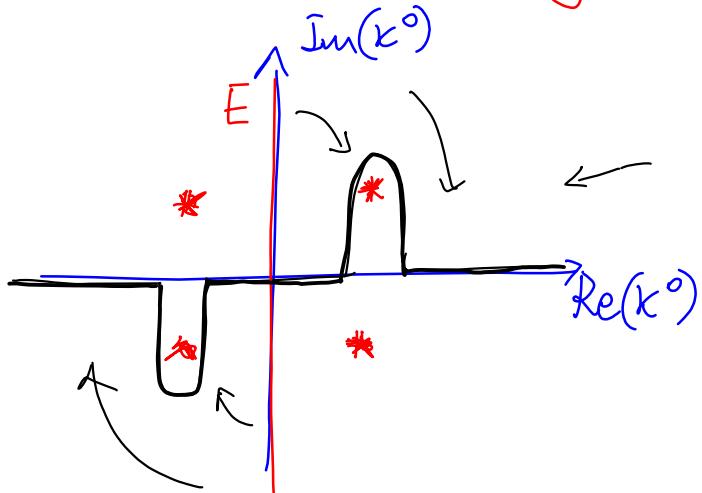


but they do not cancel in the Minkowski case



So, the Wick rotation of the Euclidean version
is the only possibility to make sense of the
HD theory from the mathematical pov

This leads directly to the Lee-Wick models



Lee-Wick contour
prescription for
the integral $\int \frac{dk^0}{2\pi}$
on the loop energy

The Lee-Wick models have been surrounded by skepticism (Lorentz invariance & analyticity are in jeopardy)

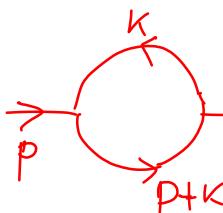
The LW prescription is ambiguous.

Additional prescriptions have been proposed,

e.g. R.E. Cutkosky, P. Landshoff, D.I. Olive, J.C. Polkinghorne, A non-analytic S matrix,
Nucl. Phys. B12 (1969) 281

but did not remove the ambiguities completely and did not uncover what is going on

The Wick rotation itself is highly nontrivial (and not analytic), but ultimately helps understanding how to correctly formulate the models

Bubble again :  = $\int \frac{dk^0}{2\pi} \int_{LW} \int_{R^3} \frac{d^3 k}{(2\pi)^3} S(k) S(p+k) =$

$$= \int_{R^3} \frac{d^3 k}{(2\pi)^3} f(\vec{k}, p)$$

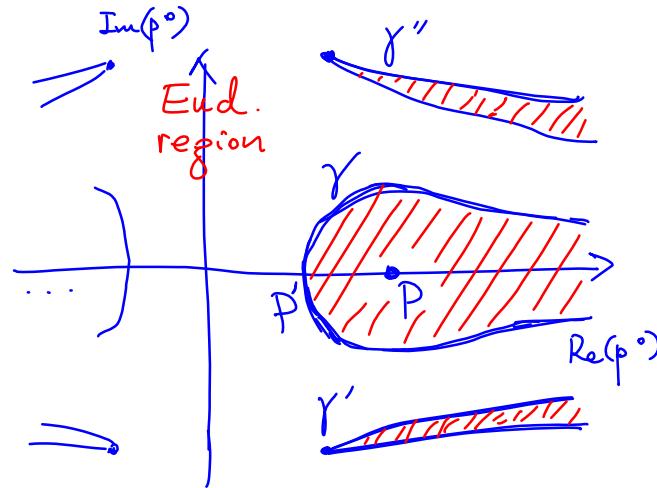
Problems where
 $f(\vec{k}, p)$ is singular

$$p^0 = \sqrt{\vec{k}^2 \pm iM^2} + \sqrt{(\vec{p}-\vec{k})^2 \pm iM^2}$$

Singularities of $f(\vec{k}, p)$:

$$p^0 \in \mathbb{C} \quad \vec{p} \in \mathbb{R}^3$$

$$p^0 = \sqrt{\vec{k}^2 \pm iM^2} + \sqrt{(\vec{p}-\vec{k})^2 \pm iM^2}$$



Inside γ the result is NOT analytic & NOT Lorentz invariant

Beyond Lee-Wick:

$$\int_{\mathbb{R}^3} \frac{d^3 \vec{k}}{(2\pi)^3} f(\vec{k}, p)$$

The problem is here!

$$= \int \frac{dk^0}{2\pi} \int \frac{d^3 k}{(2\pi)^3} S(k) S(p+k)$$

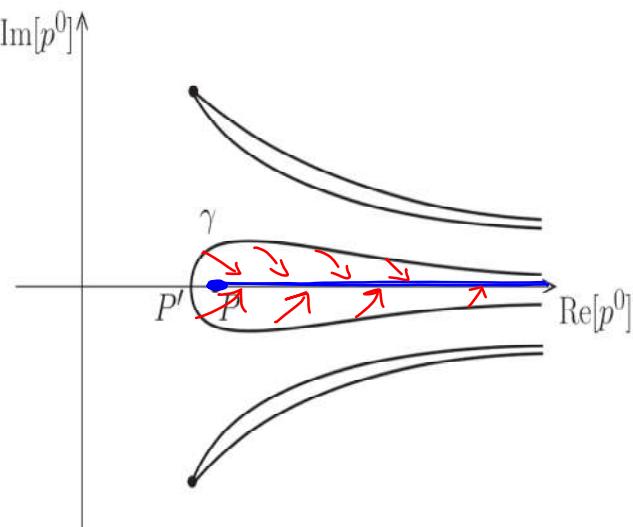
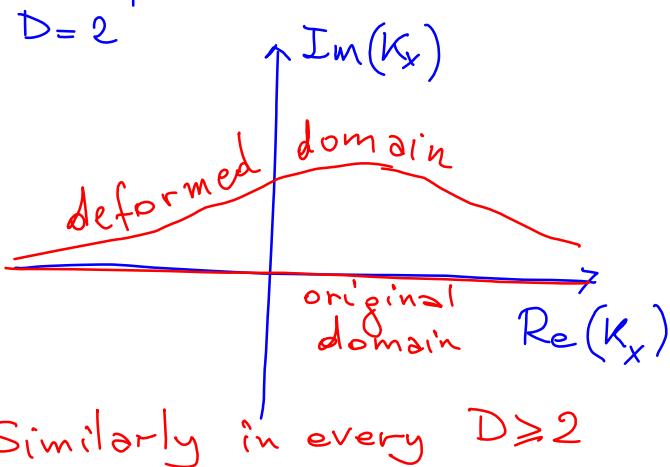
~~D_3~~

$D_3 \rightarrow$ deformed domain on the loop
(complexified) space momenta

D. A. fakeons and Lee-Wick models,

J. High Energy Phys. 02 (2018) 141 and arXiv:1801.00915 [hep-th]

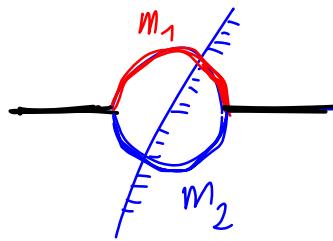
Example :



After the deformation :

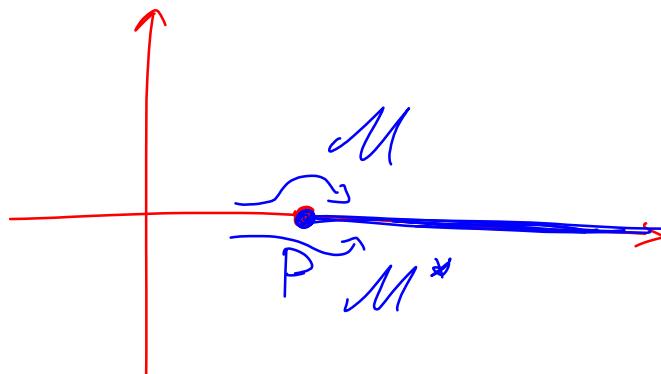
$$\int \frac{d^3 k}{(2\pi)^3} f(\vec{k}, p)$$

D_3



$$P: p^2 = (m_1 + m_2)^2$$

Optical
theorem (?)

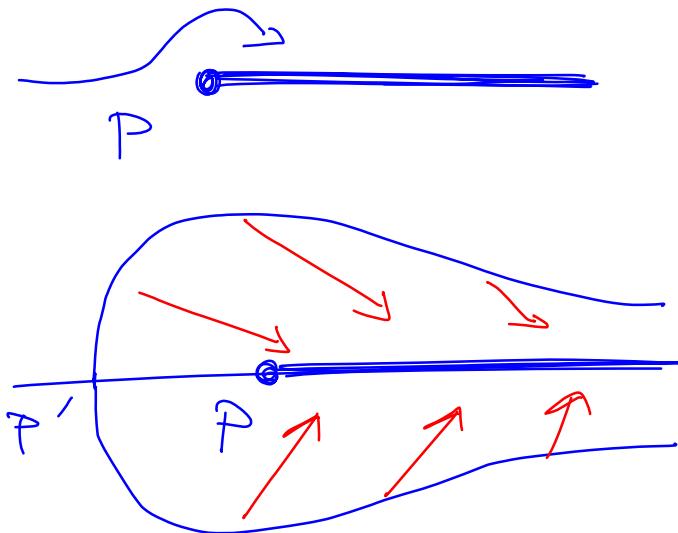


$$2 \operatorname{Im} M = \int d\vec{u} \left| \begin{array}{c} m_2 \\ m_2 \end{array} \right|^2$$

phase
space

M violates unitarity : P is the threshold
of an unphysical process $m_1^2 = \pm i M^2$ $m_2^2 = \pm i M^2$

New possibility : instead of doing

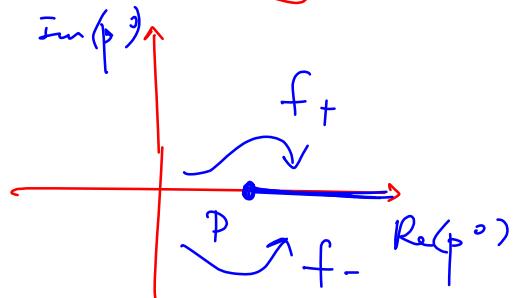


do as follows :

calculate inside,
then deform the
domain to D_3

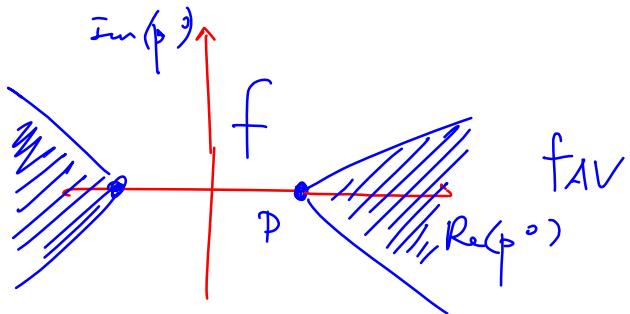
Analyticity and Lorentz invariance are recovered at the end of the deformation,
but NO imaginary part, no process ("fakreon")

The result is the arithmetic average of the two analytic continuations



$$\text{Result: } f_{AV} = \frac{f_+ + f_-}{2}$$

Analyticity & Lorentz invariance are recovered. The complex ϕ^0 plane is divided into disjoint regions of analyticity



f_{AV} has no absorptive part, so the optical theorem holds:

$$2 \operatorname{Im} M = \int d\pi \left| \begin{array}{c} f \\ f' \end{array} \right|^2 = 0$$

phase space

$\operatorname{Im} M = 0 \Rightarrow$ we can consistently project away the final states f, f' of squared masses $\pm iM^2$ from the physical space

The ideas can be generalized to non Lee-Wick models, by defining a new (real!!) quantization prescription

D. A., On the quantum field theory of the gravitational interactions,

J. High Energy Phys. 06 (2017) 086 and arXiv:1704.07728 [hep-th]

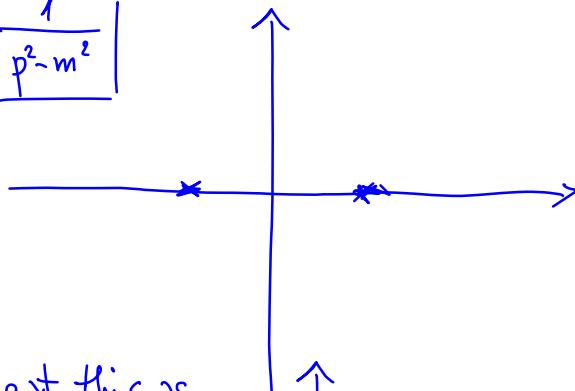
$$\frac{1}{p^2 - m^2} \dashrightarrow \frac{p^2 - m^2}{(p^2 - m^2)^2} \dashrightarrow \frac{p^2 - m^2}{(p^2 - m^2)^2 + \varepsilon^4}$$

At the end:
 $\varepsilon \rightarrow 0$

LW +
 domain
 deformation
 (or average
 nification)

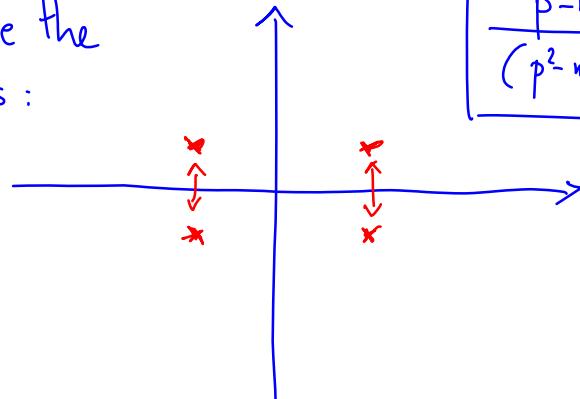
Real!

$$\frac{1}{p^2 - m^2}$$

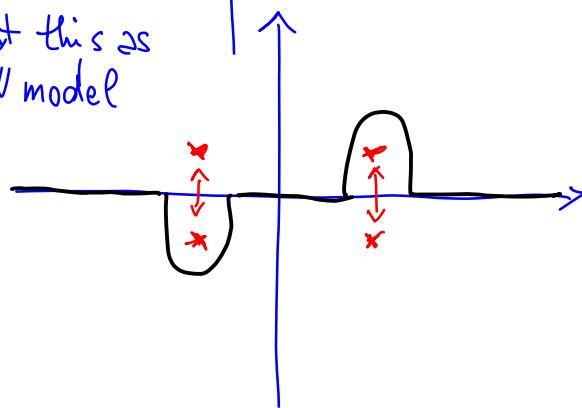


double the
poles:

$$\frac{p^2 - m^2}{(p^2 - m^2)^2 + \epsilon^4}$$



treat this as
a LW model



[Not the principal
value!]

Since the prescription is
real, there is no absorptive \rightarrow
part, i.e. no particle

CONSISTENT
PROJECTION

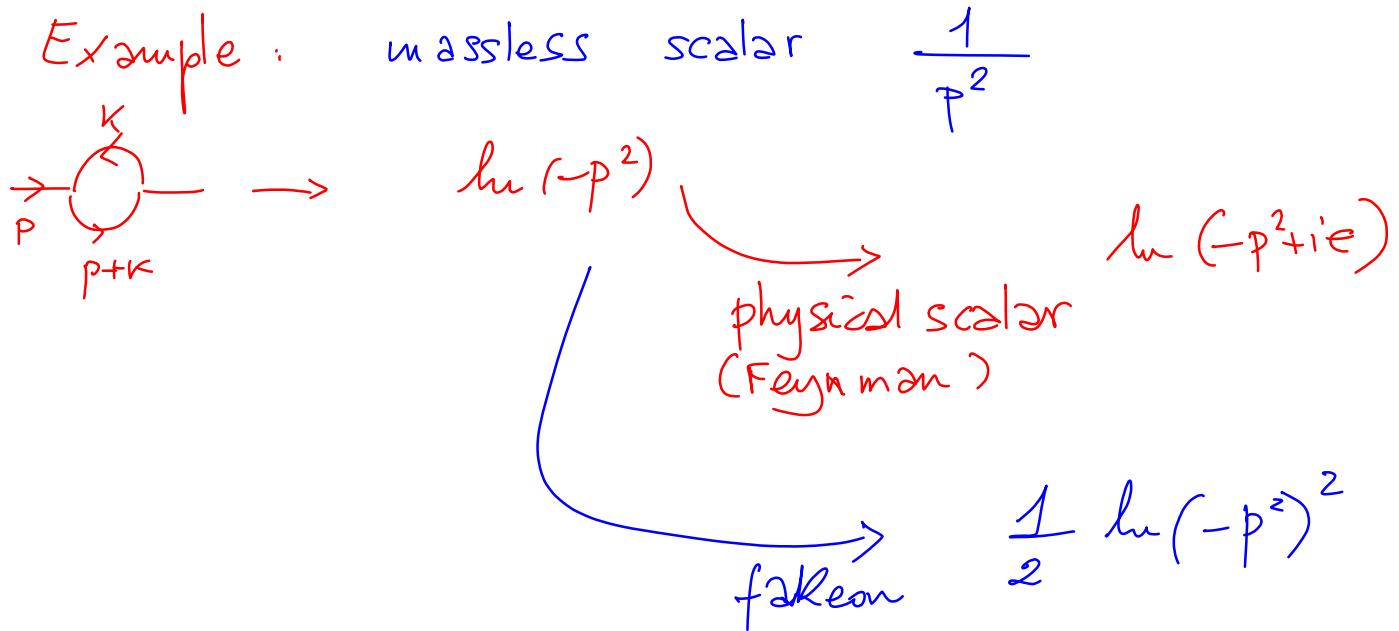
Projection:

do not use \hat{a}_f^+ of the fake particle f

$(\hat{a}_f^+)^{n_f} (\hat{a}_{ph}^+)^{n_{ph}} |0\rangle$: total Fock space \mathcal{W}

$(\hat{a}_{ph}^+)^{n_{ph}} |0\rangle$: projected Fock space \mathcal{V}

The free Hamiltonian is bounded from below in \mathcal{V}



We can cure the ghosts of

$$S_{\text{HD}} = -\frac{\mu^{-\varepsilon}}{2\kappa^2} \int \sqrt{-g} \left[2\Lambda_C + \zeta R + \alpha \left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2 \right) - \frac{\xi}{6}R^2 \right]$$

by turning them into faketons

Graviton multiplet : $\{h_{\mu\nu}, \phi, X_{\mu\nu}\}$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2k h_{\mu\nu}$$

fluctuation of the metric

massive scalar

spin-2
fakeon of mass m_X

Fakeon width :

$$\Gamma_X = -\alpha_X C m_X$$

$$C = \frac{N_S + 6N_f + 12N_v}{120}$$

$\Gamma_X < 0$: causality is violated by $X_{\mu\nu}$

$$\alpha_X = \left(\frac{m_X}{M_{Pl}}\right)^2$$

D. A. and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

D. A. and M. Piva, Quantum gravity, fakeons and microcausality, arxiv:1806.03605 [hep-th]

$$-\frac{i}{p^2 - m^2 + i\Gamma}$$

dressed falcon propagator
($\chi_{\mu\nu}$)

$$\text{Im} \left[(-i) \text{ } \begin{array}{c} \diagup \\ \diagdown \end{array} \right] = \frac{-m\Gamma}{(p^2 - m^2)^2 + m^2\Gamma^2} \geq 0 \quad \underline{\text{or}}$$

Causality :

$$\frac{i}{\epsilon - m + i\frac{\Gamma}{2}} \rightarrow \text{sgn}(t) \theta(\Gamma t) e^{-i\epsilon t - \frac{\Gamma t}{2}}$$

$\Gamma < 0$ picks the future instead of the past

Summarizing

- Positive norms
- Hamiltonian bounded from below
- Perturbative unitarity (optical theorem)
(at $\Lambda_c = 0$)
- Renormalizability, locality, uniqueness
- Lorentz invariance, general covariance
- Causality, but NOT microcausality

- Analyticity in the Euclidean region
 - + Average Continuation

= regionwise analyticity
- No asymptotic freedom
- No positive definiteness in Euclidean space